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Dangerous situations within the framework of the Nagel–Schreckenberg model

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Abstract

This paper investigates the occurrence of dangerous situations (DS) within the framework of the Nagel–Schreckenberg model. The conditions of the DS are modified. It is shown that when $v_{\max} = 1$, there will be no DS in both deterministic and non-deterministic cases. The situation is different for $v_{\max} > 1$. We show that in the deterministic case, the probability of DS covers a two-dimensional region, which depends on both the density and the initial configuration. As for the non-deterministic case, our results are qualitatively the same as previous ones, but are quantitatively different.

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1. Introduction

In the last few decades, traffic problems have attracted the interest of a community of physicists [1–3]. To understand the behaviour of traffic flow, various traffic flow models have been proposed and studied, including car-following models, cellular automaton (CA) models, gas-kinetic models and hydrodynamic models [4–11]. Compared with other dynamical approaches, CA models are conceptually simpler, and can be easily implemented on computers for numerical investigations. This allows the flexibility to adapt complicated features observed in real traffic.

The Nagel–Schreckenberg (NS) model is a basic model of traffic flow [5]. It is defined on a one-lane road of L sites with periodic boundary. Each site may either be empty or be occupied by one car. The number of cars N is conserved and each car has an integer velocity v between 0 and the speed limit $v_{\max}$. Let d be the empty sites in front of a car, the configuration of N cars is updated by four consecutive rules as the following.

(1) Acceleration: $v \to \min(v+1, v_{\max})$. (2) Slowing down: $v \to \min(d, v)$. (3) Randomization: If $v > 0$, then $v \to v - 1$ with probability p_1. (4) Motion: the position of a car is shifted by its speed v.

These four update rules are applied in parallel to all cars. Iteration over these simple rules already gives realistic results such as the spontaneous occurrence of traffic jams, the relation between traffic flow and traffic density (the so-called fundamental diagram) and the back-travelling start–stop waves.

In the NS model, car accidents will not occur. We can get it from the second rule which is designed to avoid accidents; the driver respects the safety distance. However, recent studies point out that dangerous situations (DS) exist within the framework of the NS model [12–14]. Under the DS, there will be no accident if every driver is careful enough. Nevertheless, if the drivers are not so careful ($p_2 > 0$, see section 2), the accident may occur.

This paper investigates the issue of DS in details. It is shown that the conditions of the DS in previous works should be modified. In the next section, after a brief review of the previous researches, some new findings are reported.

2. Dangerous situations

The investigation of DS begins from Boccara *et al*, who reported their numerical work in a special deterministic case, i.e. p_1 is set to 0 and $v_{\max}$ is set to 3 [12]. They assume that the drivers will probably not respect the safety distance if the speed of the car ahead $v(i+1,t)$ was positive at time t, because the drivers expect the speed of the car ahead $v(i+1,t+1)$ to remain positive at time $t+1$. Thus, a new rule (3′) is added between the NS rules (3) and (4): (3′) If $v(i+1,t) > 0$, then $v \to v+1$ with probability p_2. It is clear that the careless driving rule (3′) will probably result in an accident if the speed of the car ahead $v(i+1,t+1)$ at time $t+1$ becomes zero.

Based on the assumption, Boccara *et al* argue that when the three conditions (i) $0 \leqslant d \leqslant v_{\max}$, (ii) $v(i+1,t) > 0$, (iii) $v(i+1,t+1) = 0$ are satisfied then car i will cause an accident at time $t+1$, with a probability p_2. In other words, when the three conditions (i)–(iii) are satisfied, the car is in a dangerous situation. If the driver is careful enough ($p_2 = 0$) in this DS then there is no accident. Nevertheless, if the driver is not so careful ($p_2 > 0$), the accident may occur.

Later Huang and his co-workers extended the analysis to general situations [13, 14]. They have investigated this issue under different values of p_1 and $v_{\max}$. Moreover, a mean field analysis in the case of $v_{\max} = 1$ has been carried out [14].

However, will the car i really cause an accident at time $t+1$ with a probability p_2 when the three conditions (i)–(iii) are satisfied in the deterministic case? (i) First, we suppose that in the initial configuration, $v(i,t) < d-1$ and $d < v_{\max}$. For such a configuration, it is clear that $v(i,t+1) \leqslant d$ after the update rules (1), (2), (3), (3′), (4), which means the car accident will not occur at time $t+1$. (ii) We suppose $d = v_{\max}$, then a traffic accident will imply that $v(i,t+1) = v_{\max}+1 > v_{\max}$, which is not realistic (see also [13]).

Thus, the condition (i) must be modified into $v(i,t) \geqslant d-1$ and $0 \leqslant d < v_{\max}$. The modification guarantees that the car i causes an accident at time $t+1$ with a probability p_2 when the three conditions (i)–(iii) are satisfied in the deterministic case.

We denote P_{ac}^1 as the probability that the three conditions (i)–(iii) are met, P_{ac}^2 as the probability that the car is in a DS, P_{ac} as the probability that the car is involved in an accident. It is obvious that $P_{ac}^2 = P_{ac}^1$ and $P_{ac} = P_{ac}^2 \times p_2$ in the deterministic case.

As for the non-deterministic case, the stochastic braking of the drivers should be considered. Thus, when the three conditions (i)–(iii) are satisfied, the car will cause an accident not with a probability p_2, but with a probability $p_2 \times (1-p_1)$. Namely, $P_{ac}^2 = P_{ac}^1 \times (1-p_1)$ and $P_{ac} = P_{ac}^2 \times p_2$.

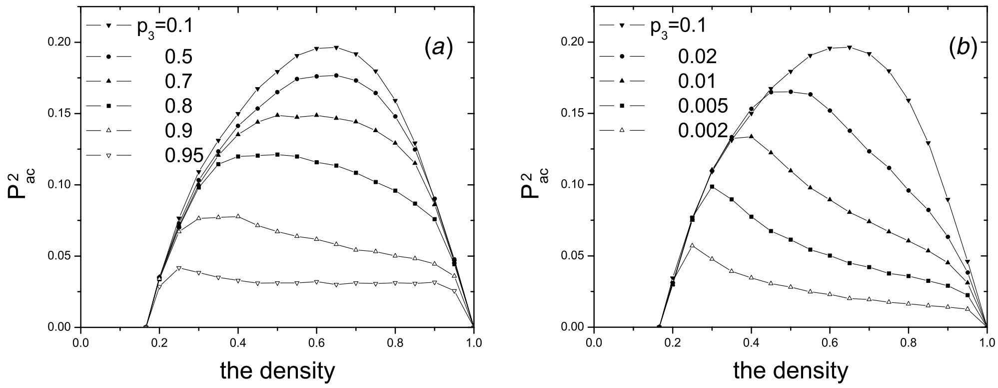

Figure 1. The dependence of P_{ac}^2 on the density and the initial condition in the deterministic case. The speed limit $v_{\max} = 5$.

Next we carry out the simulations. We focus on P_{ac}^2. As pointed out by Huang and Tseng [14], the car accident defined as a car that hits the car ahead, does not really happen in the numerical simulations. We are looking for those DS on the road and take them as an indicator of the occurrence of car accidents. In the simulations, it is the NS model that is used, the velocity increase due to rule (3′) is actually not carried out.

First, we neglect the stochastic driving behaviour and study P_{ac}^2 in the deterministic case. The simulations are carried out under different initial configurations, which are prepared as follows. We assume that at $t = -t_0$, the traffic is a megajam. Then the cars move obeying the non-deterministic NS model with the stochastic randomization p_3.[1] The system evolves from $t = -t_0$ to $t = 0$ and the traffic condition of the system at $t = 0$ is used as the initial configuration. From $t = 0$, the system will evolve according to the deterministic NS model.

We show the results in figure 1 in the case of $v_{\max} = 5$. In the simulations, $L = 1000, t_0 = 20\,000$ and the data from $t = 0$ to $t = 10\,000$ are discarded to let the transient time die out. An average over 100 different random seeds is taken for each data point[2].

From figure 1, one notes that when the density ρ of the system is below a critical density ρ_c, no DS will occur. This is because, below the critical density, there are no stopped cars.

Then, one can see that for $\rho > \rho_c$, P_{ac}^2 covers a two-dimensional region. It depends not only on the density of the system but also on the value of p_3. At a given p_3 that is not so large, P_{ac}^2 first increases with ρ, then it decreases with ρ after a maximum is reached. The maximum as well as the density for which this maximum is reached depends on p_3. When p_3 is large ($p_3 = 0.95$ in figure 1(*a*), see also $p_3 = 0.9$ and 0.95 in figure 2(*a*)), the curve of P_{ac}^2 against ρ gradually transforms into a bimodal one.

We change the value of $v_{\max}$, and find that similar results may be obtained for $v_{\max} > 1$. The simulations show that with the decrease of $v_{\max}$, ρ_c increases (see figure 2). As for $v_{\max} = 1$, P_{ac}^2 is always zero whatever ρ and p_3 are. This is easy to understand. The condition (i) requires that $d < v_{\max} = 1$, so $d = 0$. Since $d = 0$ implies that $v(i+1, t) = 0$, the three conditions (i)–(iii) cannot be satisfied simultaneously. Therefore, there is no DS.

Why can the different initial conditions lead to different probabilities of DS? To investigate the question, we show the velocity profiles of two typical initial conditions at

[1] Here we emphasize that p_3 is used only for the generation of initial conditions.

[2] Since the random function is used in the FORTRAN program, we can have slightly different results even if the density ρ and p_3 are the same provided the random seed is different.

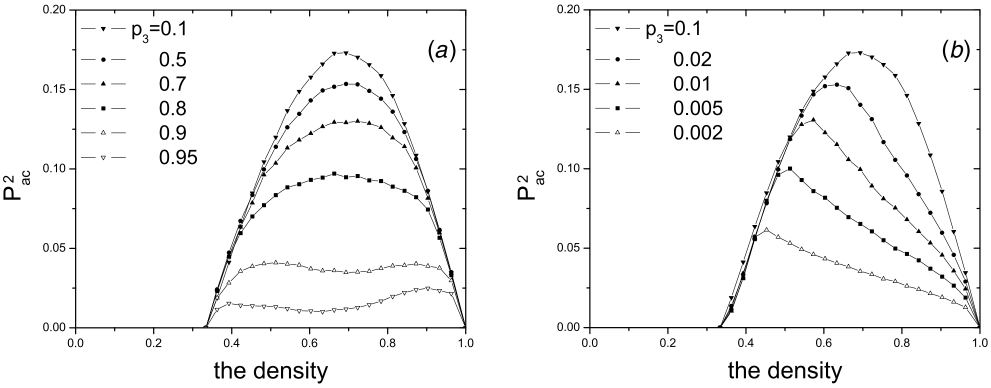

Figure 2. The dependence of P_{ac}^2 on the density and the initial condition in the deterministic case. The speed limit $v_{\max} = 2$.

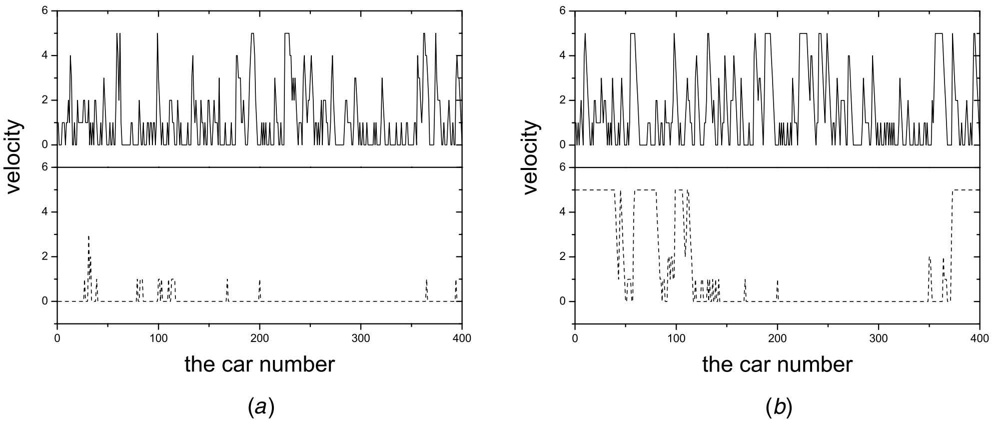

Figure 3. (*a*) Two typical initial velocity profiles; (*b*) the stationary states evolved from (*a*).

$t = 0$ in figure 3(*a*). The solid line is obtained from $p_3 = 0.2$ and the dashed line from $p_3 = 0.95$. Then after 10 000 time steps of evolution (transient time), both systems reach the stationary states as shown in figure 3(*b*). The simulations reveal that the average speeds of the two systems in figure 3(*b*) are the same, but their velocity profiles are quite different. The profile from $p_3 = 0.2$ fluctuates much more frequently than that from $p_3 = 0.95$. This more frequent fluctuation results in the higher DS probability.

We should also note that the fluctuation involves zero speed. The fluctuation between different positive speeds does not lead to DS. This can be proved by the example below.

We suppose that in the initial condition ($v_{\max} = 5$), the car with odd number has velocity 4 and gap 5 to the preceding car, and the car with even number has velocity 5 and gap 4. It is clear that this state is a stationary one. The fluctuation of the velocity profile is frequent, but there is no DS.

In this example, $\rho = 2/11 > \rho_c$. So we can conclude that under certain initial conditions, even if $\rho > \rho_c$, a DS does not occur. Another obvious example is that homogeneous traffic of any density will not lead to DS.

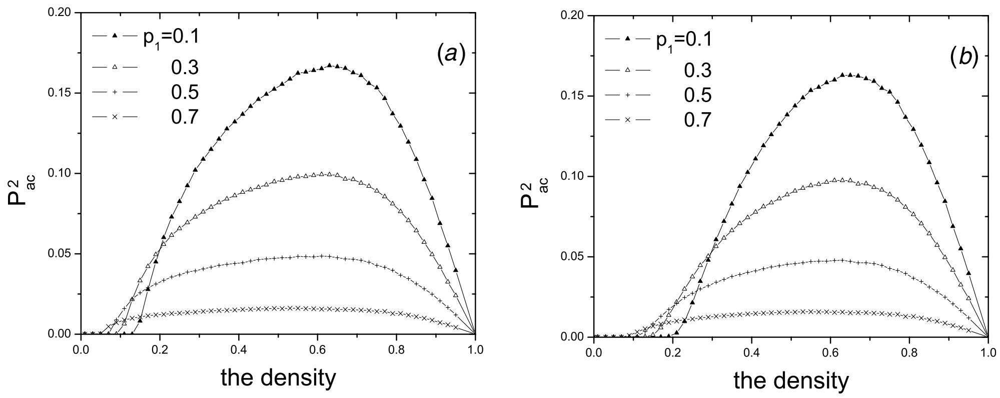

Figure 4. The dependence of P_{ac}^2 on the density in the non-deterministic case. (*a*) $v_{\max} = 5$; (*b*) $v_{\max} = 3$.

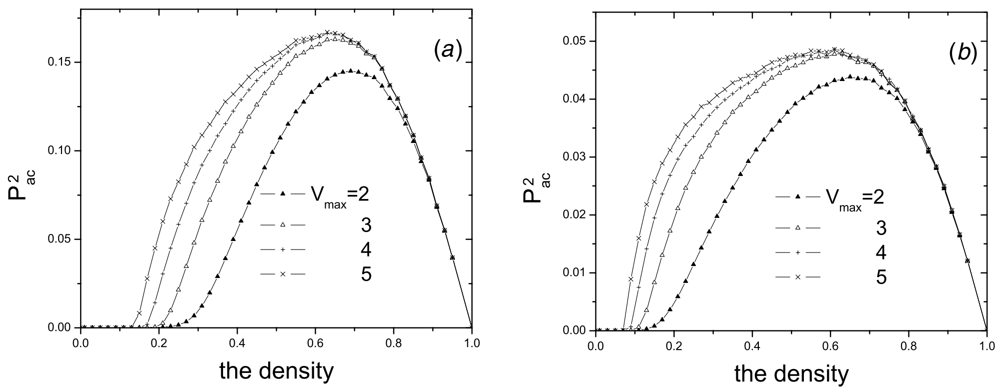

Figure 5. Probability of P_{ac}^2 as a function of the density for various $v_{\max}$. (*a*) $p_1 = 0.1$, (*b*) $p_1 = 0.5$.

In the previous works, the two-dimensional distribution of DS was not discovered. We argue that this is because the various kinds of initial distributions were not investigated in detail, i.e. the random configurations have not been classified.

Next we proceed to the study in the non-deterministic case. The simulations show that the results can also be classified into two types: (i) $v_{\max} = 1$ and (ii) $v_{\max} > 1$. In the case of $v_{\max} = 1$, there is still no DS.

When $v_{\max} > 1$, different from the deterministic case, P_{ac}^2 is independent of the initial configuration (see figure 4). Similar to the deterministic case, when the density is below a critical density ρ_c', there is no DS. For a given $v_{\max}$, $\rho_c' < \rho_c$. Moreover, both the maximum value of P_{ac}^2 and the critical density ρ_c' decrease with increasing p_1 for a given $v_{\max}$.

In figure 5, we show the probability P_{ac}^2 as a function of density for various $v_{\max} > 1$ under a given p_1. One can see that the critical density ρ_c' decreases with the increase of $v_{\max}$. Moreover, P_{ac}^2 increases as $v_{\max}$ increases in the low density region but is independent of $v_{\max}$ in the high density region. These features are qualitatively consistent with those reported previously. Nevertheless, from a quantitative point of view, the difference is obvious (see figure 6). This is not only because stochastic braking is not considered in Huang and

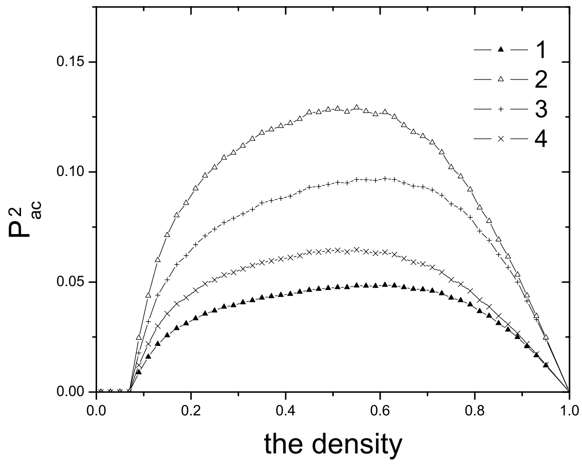

Figure 6. Probability of P_{ac}^2 as a function of the density from different works. The probability $p_1 = 0.5$, $v_{\max} = 5$. Curve 1 is our result, curve 2 is Huang and Wu's result, curve 3 is based on modified conditions (i)–(iii) without taking the stochastic braking into account, curve 4 takes the stochastic braking into account based on Huang and Wu's work.

Wu's paper[3], but also because the conditions (i)–(iii) in our work are stricter than those in Huang and Wu's work [13].

It is well known that in the non-deterministic NS model, when the density is larger than the critical density ρ_c', start–stop waves occur and the traffic flow is a coexistence of jam and free flow. Our simulations show that P_{ac}^2 is large in jam regions and it is zero in free flow regions. This is easy to understand. In free flow, there is no stopped car, so the conditions (i)–(iii) are never fulfilled. As a result, $P_{ac}^2 = 0$. In contrast, there are quite a few stopped cars in a jam, so the probability that the conditions (i)–(iii) are fulfilled is enhanced. Thus, P_{ac}^2 is large. This phenomenon is consistent with real traffic: our daily experiences tell us that under some traffic situations accidents are likely, while under other situations they are not. This result suggests that the driver should be more careful when he is in or is approaching jams.

3. Summary

In conclusion, we have investigated the occurrence of DS within the framework of the Nagel–Schreckenberg model. The conditions of the DS are modified to obtain accurate results. It is shown that for the case of $v_{\max} = 1$, there will be no DS in either the deterministic or the non-deterministic case. For $v_{\max} > 1$ and in the deterministic case, the probability of DS covers a two-dimensional region, which depends on both the density and the initial configuration. A reason for the two-dimensional distribution has been proposed. We also study the DS in the non-deterministic case for $v_{\max} > 1$. Our results are qualitatively the same as previous ones, but are quantitatively different.

Acknowledgment

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[3] In Huang and Wu's paper, P_{ac}^2 is mistakenly assumed to be the same as P_{ac}^1 in the non-deterministic case.

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